The application of Game Theory and NSGA-II algorithm to allocate resources in Emergency Management

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Abstract

When emergencies such as natural disasters or pandemics happen, it is clear that effective decision-making is critical for equitable and optimal allocation of resources. If there are more demands than resources available, it will cause many conflicts between emergency centers such as how to respond equally to locations or conflicts between allocation decision-makers. To solve that problem, we provided an alternate instruction to existing balanced resource allocation processes using Game theory. In this paper, the ideal model that we selected is the Unified Game-based model. Based on Game theory, this research proposes a non-cooperative game model for resource allocation and provides algorithms to compute Nash equilibrium. With the application of Game theory, when Nash equilibrium occurs, each player obtains an optimal strategy that leads to an efficient allocation after considering the opponent's strategy. Additionally, the Non-dominated Sorting Genetic Algorithm (NSGA-II) is also applied. Using a particular kind of crossover and mutation to create children, this algorithm then selects the following generation using comparisons of crowding distance and nondominated-sorting. The experimental results of this study show the possibility of optimizing resource allocation for emergency management sites. Keywords: Game theory, Nash equilibrium, Unified Game-based Model, NSGA-II algorithm.
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1 Introduction

This study assumes that emergencies occur in the same region and occur at once. When emergencies occur, to minimize the consequences, saving and response activities need to be carried out immediately. At this time, resources such as food, medicine and human resources will become extremely scarce and available resources in this city will become in short supply and cannot meet demand. In that case, the emergency management center needs to allocate resources appropriately [1]. Resource allocation can affect the majority of people living in disaster areas. If resources are not properly distributed, as a result, there will be places that have a lot of supplies or necessities. However, there are also places that lack them, which will make rescue more difficult and more expensive. Therefore, a fair, unbiased and consistent resource allocation system is needed. Here are some conflicts of this problem that need to be solved:

Conflict	Description
The conflict in the resources	Medical. Goods (food, water, clothes, stuff) Manpower. Finance.
The conflict between decision-makers	Between location and emergency management center. Between the different needs of the recipient. Between the receiver's different goals. Between the distances of each location. Between the severity of each location. Between the transportation speed for each location.
The conflict in planning	Between resource allocation planning. Between changes in the plan. Between fees to maintain the plan.
The conflict over the allocation of resources to priority locations	Between prioritization of locations. Between assigning manpower to perform tasks. Unable to identify unexpected situations.
The conflict in expense	Allocates distribution costs. Financial management when changes occur. Between ensuring the source of goods but still saving costs.

Table 1: The conflicts of allocating resources in emergency management [2]

Game theory has been applied to many fields, such as operations research, economics, politics, finance, conflict analysis, and so on. According to D.Fudenberg and J.Tirole [3], Game Theory is known well as the study of mathematical models applied to non-cooperative games in which many players take part, and each decision of each player is given towards their competitors' decision based on their predictions of opponents' choices.
In 1950, Nash extended the game theory by proposing a new term that came to be widely used and known as "Nash equilibrium", stating that a player can

reach his expected result just by forecasting and considering the strategies of other agents [3].
Based on the Game theory and Nash equilibrium as introduced above in the role of being the tools for resources analyzing distribution in urgent situations, this study introduces a non-cooperative game model with locations as players. Many possible strategies are used in order to fight supply shortages. The Nash equilibrium is considered to be used for computing optimal solutions and determining the supply for places. Besides, the NSGA-II algorithm is applied to improve the Nash equilibrium's effectiveness in order to distribute resources effectively in emergency situations.

2 Literature review

Resource allocation is a vital concern in emergency management. For this reason, there are various studies related to this point. Frequently, the resource allocation is solved by applying different algorithms such as exhaustive algorithm, deterministic algorithm, or metaheuristic algorithm. The majority of solutions are often based on the particular characteristics of each problem. For example, Fei Teng et al. utilized the exhaustive algorithm [3], while Morton applied the deterministic algorithm [4]. About the effectiveness of the above algorithms, Shweta Varshney and Sarvpal Singh [5] showed that most deterministic algorithms are superior to exhaustive algorithms in the experimental method. Accordingly, deterministic algorithms are not effective in distributed data conditions, making them incongruent for booking issues in the conditions which have the delay. Meanwhile, it is important to have the capacity of extension and the capacity to meet high customer conditions in emergency management. This is a climate with distributed data. Likewise, it is possible to move toward the issue of making plans for emergency locations in a metaheuristic direction like Ant Colony Optimization is feasible. The reason for this is this algorithm can give almost ideal outcomes in a sufficient time. Over the most recent couple of years, numerous resource allocation problems were coped with game theory in emergency management. Scaparra , Church [6] concentrated on the effect of a tragedy on supplement substructure, what tends to be allowed offices that give labor and products to an area that is impacted by catastrophe. One lack of an office in view of a natural error increases activity costs and diminishes organization. Scaparra and Church protected the infrastructure with open assets by a "tri-level" streamlining model and a strategy relating to a tree search plan to ease the impacts of a disaster. Because of the tight assurance, assets are accessible for disaster reaction in the two cases above. The non-cooperatives were investigated for the reaction time, and cost to use action in project booking by Vásquez et al. [7]. Players exercise to limit the responsibility of assets. They were persuaded to deal with a crisis reaction project booking issue. Different papers likewise center around project booking endeavors to limit how much chance to arrive at a crisis, while amplifying the effect of crisis help is Smyrnakis and Leslie's paper [8]. They applied a stochastic nonexistent play model to the reaction time of disaster the board. They looked at the response of ambulances around to hurt people and the reaction time and practicality they have. Moreover, it has two papers dissecting departure models as well as fruitful clearances from structures with restricted exits. Lo et al. [9] utilized non-helpful and complete data games while Zheng and Cheng [10] applied agreeable and serious games. Contrasting two papers, with a bigger impersonation impact and increased collaboration, it viewed departure times as longer, capability may not be guaranteed with participation. Notwithstanding, Bouzat and Kuperman [11] model passerby room departure with game hypothetical models. They found that in spite of the way that leaving gives a reasonable choice to the players, given express conditions normal collaboration can transform into the best methodology. Balancing and placement problems of the emergency locations were explored by Ye and Chen [12]. They were built on noncooperation. They attended to the Nash equilibrium and do not concerned much about another way for an optimal allocation strategy. In a study of R.Minciardi et al. [13], numerous mathematical formulations in programming were applied, giving the decision makers a clear view in which strategies will be ideally chosen to allocate resources when facing peril disasters caused by nature. They clearly stated two stages of this emergency, entailing pre-event and during the event, built a specific scenario and then made a comparison between these strategies' performances. A model of game with information comprehensively declared is also represented in the paper of Wang et al. [14], in which the NSGA-II mathematical methods are used for accomplish the Nash Equilibrium. The players are brought into play to cope with the crisis in which each area has a desire to reach as many resources as possible. On the other hand, these papers just concentrate on adequate general objectives in the whole world and have not paid attention to either specific location or area yet. Besides, limited resources leading to social impartiality are not mentioned, each area has its own characteristics so one general scenario cannot be applied to all. In order to find the key to dispensing emergency resources, many scholars have built programming models to deal with appropriate distribution plans. For instance, Y.He [15] designed a prediction tool to predict the therapeutic needs in locations suffering from disasters by using Susceptible-Exposed Infectious-Recovered (SEIR) model and then allocated throughout linear programming. In view of the fact that many players take part in the game with altered goals, many researchers have put related locations that are in the act of resource distribution for urgent situations into a simulation. Youwei Yuan et al. [16] studied the supplies in hospitals by developing a noncooperative game model and proved that this model can narrow the costs down. C.Binley et al. [17] proposed a system for supply to healthcare providers as fair as possible when the resources are rare and in high demand.

Author	Factors
[3] Fei Teng et al.	Time-overlapped manner, center the improvement of the portion of one sort of asset
[4] Morton	Planning frameworks, beginning single machine or asset and continuing on toward assembly shops
[6] Scaparra and Church	Needed goods or services, Tri-level optimization model Facility protection
[7] Vásquez et al	Reaction time, cost to use or not use movement in project booking, assets with restricted accessibility to convey Non-agreeable game
[8] Smyrnakis and Leslie	Vehicle assignment, Scheduling systems, impact of emergency help, Stochastic, fictitious play model
[9] Lo et al.	Incomplete information, departures from working with restricted exits, Non-cooperative
[10] Zheng and Cheng	Successful evacuation and cooperation/ competition Cooperative/competitive
[11] Bouzat and Kuperman	Evacuation situation, Pedestrians, N-player game
[12] Ye and Chen	Multidimensional resource, Nash equilibrium Non-cooperative game
[13] R.Minciardi et al.	Available resources, Mathematical programming formulation Nash equilibrium

Table 2: Some highlight publications and their factors

After the examination, we observed that the above articles have a few weaknesses that destitute individuals completely settled the irregularities on crisis the executives as follows: [3], [4], [6] center on analyzing algorithms but these papers do not give solutions which apply this algorithm. In my paper, we both analyzed nsga-ii algorithm and proposed solutions based nsga-ii algorithm.; [9], [10] only focus on a single characteristic, so it isn't reasonable for the necessity of multi-objective conditions, for example, emergency management and Game Theory; calculations of [11] is just can apply to occasions where gatherings should choose the proper behavior because of a debacle occasion like a cataclysmic event. Gaining for a fact, we considered various viewpoints as well as qualities, for example, reaction time, cost, and departures. [12] involves non-helpful games and agreeable games for some qualities. Accordingly, it prompts struggle on the grounds that every issue needs another arrangement. We just utilized non-helpful games in my paper, to look at changing perspectives without any problem.
From the inadequacies of the above investigations, we had gained as a matter of fact, and chosen to apply the model based on the insect settlement streamlining calculation to give a new arrangement that can take care of the contention issues

and advance the model's exhibition.
Our research mentions a non-cooperative game template that allows participants (emergency locations), one of EL and a bunch with allocative strategies, to fight to dispense assets from the source, not entirely settled to propel their presentation. Nash harmony calculation is used as the advancement calculation for designation. The NE is viewed as a socially ideal arrangement that guarantees the same way of behaving of the players and creates a decent quality ideal arrangement. In order to get NE, and plan an increased version of nsga-ii algorithm utilizing the unique fast sorting technique. non-ruled arranging hereditary calculation II (NSGA-II) is a changed hereditary calculation that attempts to deliver non-overwhelmed answers for multi-objective improvement issues by mimicking the regular determination process. Through reproduction investigation and cross-section correlation, this algorithm can meet the emergency management environment, with more reaction time and better quality, more appropriate for the emergency management field.
Our main contributions are as follows:
(i) Based on the Unified Game-based model, propose solutions including players, their strategies, their payoff function and Nash equilibrium.
Prove the feasibility and effectiveness of the model by applying the NSGA-II mathematical methods in order to explore Nash equilibrium of this problem.
The remainder of this article is compiled as follows. We introduced issues as well as analyzed examples including citations for these problems. Next section we applied the Unified Game-Based Model to the problem mentioned above. We presented a solution with Nash equilibrium in the next section and in the last section, concluded with a summary and showed our future work.

3 Problem Description

3.1 Description of problem

Resource allocation problems in urgent cases can be simply clarified as a situation focusing on discovering how to distribute available and unchangeable resources to areas when facing emergency circumstances, while cutting down on cost is also highly considered. The main problem in other words is to determine how much of each resource to allocate to each task in emergency situations, because every locations' expectation is as many resources as possible while the supply is limited. Numerous other conflicts also appear in every phase that have been listed in Table [2]. The examined research is the course of a huge scope disaster crisis area wherein there are at least two managerial areas. Each managerial region has numerous asset supply focuses (request locales) and different asset request focuses (impacted areas).
As per Yanyan Wang (2021), to distribute crisis assets economically, a multiobjective improvement issue should be tackled to limit both the conveyance time and expenses and to expand the crisis asset designation

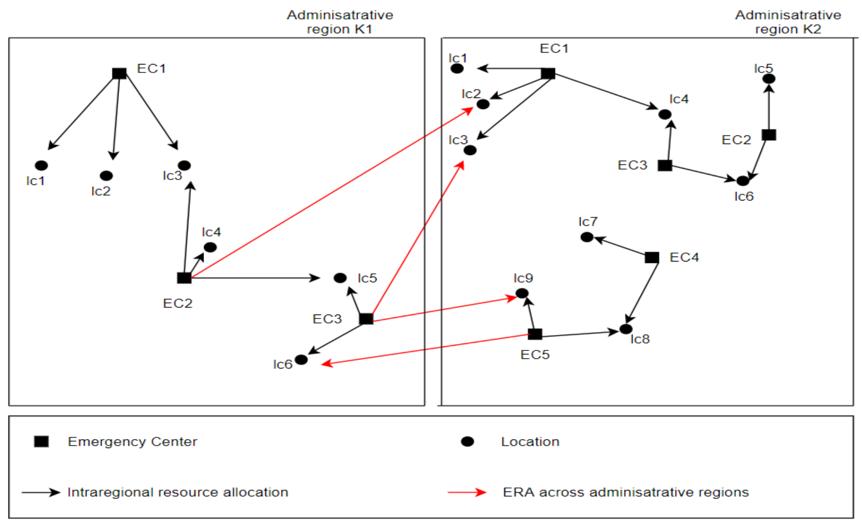

Figure 1: Schematic diagram of emergency resource allocation [18]

inclusion rate across the disaster-impacted areas. Practical crisis areas shouldn't just fulfill the asset needs of the disaster-stricken areas in the ongoing crisis period as far as possible, yet in addition guarantee that the capacity of the crisis site to address the issues of the disaster-stricken focuses won't be hurt later on in the crisis period. It alludes to making a logical crisis asset distribution intend to accomplish the multiperiod worldwide ideal interest impact. It is to figure out an asset distribution plan according to the point of view of the ideal impact of the whole crisis time frame (not just considering the asset designation plan of a specific crisis period).

3.2 Properties Characteristics

3.2.1 Assumptions

We made the following assumptions after considering emergency resource allocation according to the actual situation:
Resource Allocation Centers can capture and respond promptly to disasters at any location in the region. They can obtain information about the amount of influence locations, highway state, the injured person, the demand of reserve and accessible reserve.
Major resources in the resource center contain unrelated items. If a particular resource kind is allocated to a location, the allocation of other resource types will not be affected.

3.2.2 Data of the problem

When faced with a disaster, emergency sites cannot cope on their own and will strategically request relief packages from resource centers. Within the relief package, each method of applying for relief packages is determined by a combination of the following factors:
Demand resources of location which includes many different types and available resources of the resource center.
Disaster level (1-10).
The priority of the location compared to other locations, here will be the proportion of the people trapped compared to the whole population.
Time required for getting relief package (hours).
If multiple locations in the same area are hit by the disaster, they will have different strategies non-cooperate or cooperate.

3.2.3 Non-cooperate strategies

Emergency locations require a number of resources that exceed their level of disaster or priority.
Effect: Since the center doesn't have enough resources, it may not be able to immediately respond to the player's request so that the relief will be ranked later.
Emergency locations reduce the time to request relief as much as possible and ask the dispensing center to get to the first locations. However, this means they must reduce the amount of demand resources.
Effect: The Center may not be able to prepare resources in time or may not have enough resources to provide relief.
Hubs simultaneously send requests to the resource center closest to them.
Effect: When the center's available resources are not met, under-resourced locations weather the disaster.

3.2.4 Cooperation strategies

TThe emergency locations offer an appropriate level of resources, require the right response time and choose a suitable center after considering the options of other locations to get the benefit. Effect: High-potential centers all meet the right number of resources in the right amount of time to emergency locations. In order for the people and governments in the region to reap the greatest benefits – meaning they get a fair and efficient share of their resources; it is important to have an organization that shares their goals. In this article, we introduced the special player resource centers. When centers receive requests for relief packages from emergency locations, they need to come up with relief strategies. These strategies depend on factors such as contributed resources, fixed cost of transportation, variable cost of allocating resources and unmet resources. The model of the problem was summarized and determined in Table 3, as follow:

Player	Characteristics	Description
Special player: Resource center	*Allocated resources*	Number of resources allocated to emergency location
	Unmet resources	Number of resources has not been met with the demand resources
	Fixed cost of transportation	Fixed cost of transporting resources from resource center to emergency location
	Variables cost of allocating resource	Variables cost of allocating resource from resource center to emergency location
	Deciding on relief	Binary variable indicates whether relief is available for an emergency location (yes or no)
Normal player: Emergency location	*Demand*	Amount of demand resources which include many different types
	Available	Number of available resources in resource center
	Time	Minimum time to respond to an emergency or to receive relief package.
	Level	Catastrophic level of emergency location (1-10)
	Priority	Priority of emergency location relative to other sites (ratio of trapped population to total population)

Table 3: The characteristics of players in model

There will be conflicts occurring directly between disaster sites and between disaster sites and resource centers as soon as the amount of accessible reserve of the reserve center cannot accommodate all the requests of the locations. Conflict will be resolved if all requests are determined.

3.3 A realistic example

An example of resource allocation problems when emergencies occur is the massive flood in Kerala [19]. With extraordinary heavy rainfall, in August 2018, one of the worst floods in human history occurred. The rising water from the rivers along with a large amount of water discharged from the dams have completely paralyzed the mountainous districts. Most areas in Kerala experienced flash floods along with landslides caused by erosion. The Army, Navy, Coast Guard and volunteer units have been mobilized to carry out the rescue. However, different natural disasters occur at different places making rescue and provision of resources extremely difficult. Landslides affected the districts of Wayanad and Idukki, flooding had a devastating effect on Malappuram and Alappuzha. To solve that situation, Ezhimala and Cochin, two Naval Base Stations, were utilized for resource management and distribution to the victims. In addition, the Arakkonam station will be a backup resource station, providing resources as well as helicopters in the condition that the main two-touch is not sufficient. This is an example of a game of non-cooperative and fair resource allocation.

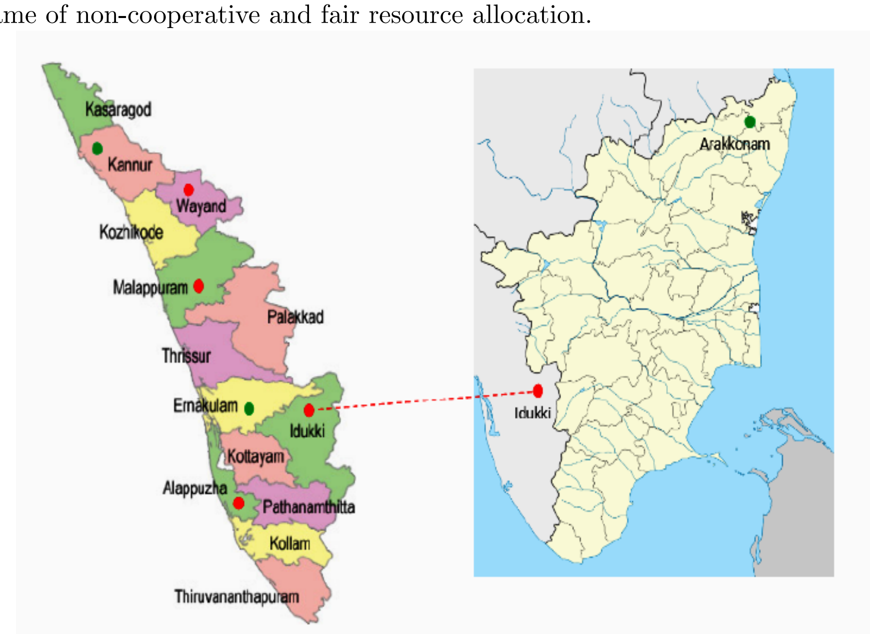

Figure 2. Disasters location (red dots) and Emergency center (green dots) [19]

3.4 A sampled data set

In the above example, if Game theory and Nash balance are not used, as a result, the number of helicopters deployed to the locations may be insufficient or excessive compared to actual needs. That can make disaster assistance and recovery more expensive and time-consuming. In addition, the fact that the number of helicopters coming to the rescue is less than necessary can also make the lives of people more difficult when they lack food or clean water. For the relief work to be most effective, a Nash equilibrium occurs when the allocation of resources is fair and the number of helicopters required must also be met. In this case, the Cochin management center will be responsible for two locations, Alappuzha and Idukki. Meanwhile, the resources of the Ezhimala station will take care of the other two areas, Malappuram and Wayanad. The table [4] below shows the sort order from resource management centers to player locations.

Players	Distance from Cochin (EC1)	Distance from Ezhimala (EC2)	Distance from Arakkonam (EC3)	Sort order in term of distance
Alappuzha	70	375	650	EC1<EC2<EC3
Idukki	100	350	500	EC1<EC2<EC3
Malappuram	186	150	560	EC2<EC1<EC3
Wayanad	200	130	530	EC2<EC1<EC3

Table 4: Distances from emergency center (EC) and distances' sort order

Assume that a helicopter is responsible for distributing resources in an area of 400 square kilometers, the number of planes required for a single location can be calculated using the following formula [19]:

$$\text{Area (square kilometers) x Criticality} \approx 400 \text{ x Helicopters} \tag{1}$$

The number of helicopters is calculated as table 5 below:

Players	Area (square kilometers)	Criticality	Helicopters
Alappuzha	1415	0.6	2
Idukki	4360	0.55	6
Malappuram	3555	0.7	6
Wayanad	2130	0.6	3

Table 5: Required number of helicopters

However, helicopters available at the Cochin and Ezhimala relief stations as shown in the table below are not enough. Therefore, the backup aid station, Arakkonam, will need to send helicopters to support the rescue of people and provide resources.

Players	Cochin (EC1)	Ezhimala (EC2)	Arakkonam (EC3)
Alappuzha(P1)	2	0	0
Idukki(P2)	6	0	0
Malappuram(P3)	0	6	0
Wayanad(P4)	0	3	0
Necessity	8	9	0
Storage	7	8	5

Table 6: Conflict between necessity and storage

From table 6 we can see that the total number of player requests has exceeded the capacity of EC1 and EC2. Since the distance between EC1 to P1 and P2 is the shortest, we prioritize the allocation of resources for these two regions from EC1. Similarly, EC2 is also used as a resource allocation station for P3 and

P4. However, the number of helicopters available at EC1 and EC2 stations is not enough to meet the needs of players. Therefore, to accommodate enough helicopters for all players, the number of spare helicopters in EC3 needs to be utilized.

3.4.1 Cost in terms of response time and fuel consumption

The helicopters used in this rescue by the Indian navy have 210km/h, besides, each helicopter to fly one kilometer needs 0.0025 kilolitres of fuel. So we have the following table 7 [19] for statistics of the time from the stations to the place of the disaster and the amount of fuel needed.

Players	Emergency center	Distance(km)	P_c (kilolitre)	P_t (hr.)
Alappuzha(P1)	Cochin (EC1)	70	0.175	0.33
Alappuzha(P1)	Arakkonam (EC3)	650	1.625	3.1
Idukki(P2)	Cochin (EC1)	100	0.250	0.48
Idukki(P2)	Arakkonam (EC3)	500	1.250	2.4
Malappuram(P3)	Ezhimala (EC2)	150	0.375	0.71
Malappuram(P3)	Arakkonam (EC3)	560	1.4	2.67
Wayanad(P4)	Ezhimala (EC2)	135	0.3375	0.64
Wayanad(P4)	Arakkonam (EC3)	530	1.325	2.52

Table 7: Fuel consumption and response time

Players will need a certain number of aircraft to be able to respond to disasters effectively. Since there are not enough helicopters at the nearest distribution centers, to efficiently distribute resources from EC1, P1 and P2 engage in a two-player game. The same thing happens when P3 and P4 play a two-player game to obtain resources from EC2. Getting resources from the nearest management center will help reduce transportation distances, thereby saving fuel and necessary costs.

3.4.2 Penalty depending on shared behaviors

The goal of players is to acquire as many resources as they can. However, this leads to a situation where the number of requests exceeds the capacity of emergency management centers. Therefore, a penalty [19] will be applied so that the players do not take all the resources available locally. In this game,

$$P_{p1} = \left[\frac{n1-d1}{n1}\right] + \frac{n1-d1}{n1} + \frac{n2-(a1-d1)}{n2} + \left[\frac{\frac{d1}{n1}}{\frac{d1}{n1}+\frac{d2}{n2}}\right] \tag{2}$$

$$\mathrm{P}_{p2} = \left[\frac{n2-d2}{n2}\right] + \frac{n2-d2}{n2} + \frac{n1-(a1-d2)}{n1} + \left[\frac{\frac{d2}{n2}}{\frac{d1}{n1}+\frac{d2}{n2}}\right] \tag{3}$$

In the above formula, n1 and n2 are the numbers of resources P1 and P2 need, d1 and d2 are the numbers of resources required from these two regions, and the total of resources that EC1 can provide is denoted by a1. In (2) and (3): The first term is that the player will be penalized when the amount of resources available is more than the total need.
Dissatisfaction with the player's choice is the second term. The player will be penalized when the required amount of resources is less than the required amount.
The third term presents the restriction of one player to the other.
The fourth term punishes player conflict and if the player requests more resources than can be accommodated, it will be non-zero.
Similarly, we have the following formula for calculating the penalties [19] for P3 and P4:

$$\mathrm{P}_{p3} = \left[\frac{n3-d3}{n3}\right] + \frac{n3-d3}{n3} + \frac{n4-(a2-d3)}{n4} + \left[\frac{\frac{d3}{n3}}{\frac{d3}{n3}+\frac{d4}{n4}}\right] \tag{4}$$

$$\mathrm{P}_{p4} = \left[\frac{n4-d4}{n4}\right] + \frac{n4-d4}{n4} + \frac{n3-(a2-d4)}{n3} + \left[\frac{\frac{d4}{n4}}{\frac{d3}{n3}+\frac{d4}{n4}}\right] \tag{5}$$

4 Model

4.1 Introduction to Unified Game-based model

The purpose of the Unified Game-base model is to create a special player and this player is different from the rest [20]. In the game related to investing, this particular player will be the investor. On the other hand, in a non-investment game, the role of this player will be as the referee. Simultaneously, the influence of the factors in each player's strategy will be shown through a feature vector. Therefore, by incorporate of three factors: special player, ordinary players and the conflicts of all players, we give an unified model as below:

$$G = (\{P_0, P_i\}, \{S_0, S_i\}, \{U_0, U_i\}, R^c \tag{6}$$

where,
G: perform the game.
P_0: the special player who can be the stakeholder or the referee to ensure the interests of all players.
$S_0 = \{S_{01}, .., S_{0j}, .., S_{0M0}\}$: special player's strategy; the quantity of the special player's strategies is M0. U0: a payoff function from the special player's strategies in real numbers.
U_0: a payoff function from the special player's strategies in real numbers.
$P_i = \{P_1, .., P_i, .., P_N\}$: normal player and N is the total of the players.
$S_i = \{S_{i1}, .., S_{ij}, .., S_{iMi}\}$: normal players' strategies; M_i is the total of the ordinary players' strategies.
U_i : a payoff function from the normal players' strategies in real numbers.
R^c : a vector representing the conflicts between the strategies of M players in problems.

4.2 Mathematical model

As introduced above, we applied the above model to solve our problem. Players in this game are emergency location i and resource center q. The Unified Game-Based Model can be described as follows:

$$\text{G} = (\{P_0,\, P_i\},\, \{S_0,\, S_i\},\, \{U_0,\, U_i\},\, R^c) \tag{7}$$

where,

P_0: Resource center - represent the benefits of the entire population and government in the disaster area.

$S_0 = \{S_{01}, .., S_{0j}, .., S_{0Z0}\}$: is set of emergency center's strategies; (Z_0: Total strategies of emergency center)

U_0: a payoff meaning which represents the emergency center's strategies P_i: Emergency location

$S_i = \{S_{i1}, .., S_{ij}, .., S_{iZi}\}$: is a set of the emergency location's sequence activities process; (Z_i: Total strategies of emergency location i)

U_i: a payoff meaning which represents the emergency location's strategies

R^c : a vector representing the conflicts.

Strategy of resource center:

Set of the emergency location's strategies, $S_0 = \{S_{01}, ..., S_{0j}, ..., S_{0Z0}\}$

A special player's S_{0j} strategy (representing the resource center) consists of the following components: Contributed resources, fixed cost of transportation, variable cost of allocating resource and unmet resources.

Strategy of emergency location:

Set of the the emergency location: $S_i = \{S_{i1}, ..., S_{ij}, ..., S_{iZi}\}$ is a set of actions to get the benefit of player i with the information to get the relief package, S_{ij} consists of the following components: cost, level, priority, time.

Payoff function of resource center

$$U_0 = (\textstyle\sum_i \sum_q F_{iq} * B_{iq} + \sum_i \sum_q V_{iq} * L_{iq}) - \sum_i P_i \tag{8}$$

In which,

Q: Set of resource centers, $q \in Q$, Q is the total number of resource centers.

F_{iq}: Fixed cost of transportation from resource center q to location i.

V_{iq}: Variable expenses for distributing relief packages from resource center q to location i.

B_{iq}: Variable that points out if resource center q distributes relief package for location i or not

L_{iq}: Amount of resource contributed by resource center q for emergency location i.

P_i: Amount of resources that is not yet met the demand of resource c for i. (P_i = M_i - L_{iq}, M_i: demand for resource in i).

The smaller U_o, the smaller the total allocating costs and the total losses caused by insufficient resources.

Payoff function of emergency location

The payoff of location i can be demonstrated as follow:

$$U_i = \textstyle\sum_i [X * (Available_i - Demand_i) * Level_i * Priority_i + Y * Time_i] \tag{9}$$

In which,

Demand: the demand resources include many different types of location i

Available: the available resources of resource center where location i send a request for help.
Level: the disaster level of location i
Priority: he priority of the location i compared to other locations, here will be the proportion of the people trapped compared to the whole population and the distance between location i and resource center compared to other locations.
Time: Minimum time required for getting relief packages of location i from resource center (hours)
X and Y are the expert constants that regulate the quality of the fitness value
The bigger the U_i, the higher the rate of getting relief packages from resource center.
The general fitness function:
The adaptive function of the project is calculated according to the payoff function of each player. If the difference between two payoff values is smaller, the solution will be better

$$\mathrm{U}_{fitness} = |A * U_o \ - \ \sum_i U_i * B| \tag{10}$$

In which, A and B are the expert constants that regulate the quality of the fitness value
Constraint:
There are some constraints that must be fulfilled, as follow:
The sum of distributed resources shall not be higher than the accessible reserve:

$$\sum_i L_{iq} \leq N_q, \ (N_q\text{: The accessible reserve in } q) \tag{11}$$

The total amount of resource allocated to all emergency locations shall not exceed the sum of the current demand and the unmet proportion remaining:

$$\sum_q L_{iq} \leq M_i + P_i, \ ((M_i\text{:The demand for resource in } i) \tag{12}$$

The binary variable have to be equal to 0 or 1, as demonstrated in

$$B_{iq} \in \{ \ 0, \ 1 \ \} \tag{13}$$

The variables for allocation, unmet of required resource have to be non-negative, as shown in:

$$L_{iq} \geq 0 \tag{14}$$

$$P_i \geq 0 \tag{15}$$

4.3 The Nash Equilibria formula, and its application

We pointed out a payoff matrix that based on the act of location i:

$$\begin{vmatrix} U_{i1}^1 & U_{i1}^2 & \dots & U_{i1}^{H(i)} \\ U_{i2}^1 & U_{i2}^2 & \dots & U_{i2}^{H(i)} \\ ... & \dots & \dots & \dots \\ U_{iG(i)}^1 & U_{iG(i)}^2 & \dots & U_{iG(i)}^{H(i)} \end{vmatrix} \tag{16}$$

Where U_{ij}^k is the benefit of location i when they pick a precise act S_{ij} as soon as other locations vote for an act S_{-ik}. S_{-ik} stands for the k^{th} mixture acts of other locations. $G_{(i)}$ is the sum of the act of location i. $H_{(i)}$ is the sum of the mixture acts of other locations.

The Nash equilibrium of the game is a strategy where no one emergency location can increase profits when the others have fixed the strategy. As soon as location i picks the j^{th} sjth, if this strategy is the optimal strategy denoted by S^*_{ij}, the optimal strategy of the other players is denoted by S^*_{-ik} then the Nash equilibrium of the strategy will obey the condition, as follows:

$$\mathrm{U}_i(S^*_{ij}, S^*_{-ik}) \geq U_i(S_{ij}, S^*_{-ik}) \tag{17}$$

4.4 Nikaido Isoda function

To broaden the scope of the Nash equilibrium issue in non-cooperative games, H. Nikaido and K. Isoda (1955) introduced the Nikaido-Isoda function, which defines the Nash equilibrium in conflict. In the resource allocation problem, the function is modeled as follows:

$$\mathrm{f(x^*,x)}=\sum_{i=1}^{n} \left(f_i(x) - f_i(x[y_i])\right) \tag{18}$$

By using the Nikaido-Isoda function, the Nash equilibrium is found when the formula 18 is satisfied [21]. In fact, the Nash equilibrium must also satisfy other factors such as the problem data and to find the Nash equilibrium according to the Nikaido-Isoda function, the problem will have a multi-objective optimal form and must be solved by multiobjective evolutionary algorithms (MOEA). Thereby, the application of the Nikaido-Isoda function shows that the resource allocation conflicts can be converged and from which the Nash equilibrium can be found.

5 Algorithm with Nash equilibrium

5.1 Reason to use NSGA-II algorithm to deal with the issue

Resource allocation in emergency problems which seeks to find the expected objectives by allocating the limited number of resources to various activities. Human, money, materials, etc are resources that are supplied limited to complete the targets. The targets such as minimizing costs or maximizing efficiency are usually promoted by specific demands in the future. As a result, the model of the problem is a multi-objective problem, requiring simultaneous consideration of the objectives, thereby finding the most reasonable resource allocation. In this paper, to obtain an array with compound efficiently, we proposed a NSGA-II. This is another version of the genetic algorithm (GA) that mimics the normal determination method. GA algorithm is used for single-objective optimization problems, NSGA-II is a hereditary calculation to tackle multi-objective enhancement issue. It is also an algorithm in the group of evolutionary computational algorithms [22]. Currently, there are many researchers applying the NSGA-II mathematical method to deal with their problems. With the NSGA-II algorithm, we found that the model performs appreciably better than other algorithms in the existing MOEA with regard to capacity and scope. This shows that it can bring more and higher quality testing time allocation schemes, especially for large, complex datasets. According to Mingjie Song and Dongmei Chen [23], NSGA-II is a multi-objective streamlining calculation that attempts to distinguish non-ruled arrangements that address different compromises between numerous goals [24]. For the previous GA algorithm, through crossover and mutation, children in the following generations are produced based on the process of randomly selecting parents from the population. On the other hand, in NSGA-II, individuals are selected regardless of their rank and crowding space. This is the difference between NSGA-II and traditional GA, which is based on the theory of elites, the mechanism of preserving diversity is clear and by not being dominated.

5.2 How NSGA-II algorithm can find Nash equilibria

In the game of noncooperation, the most basic concept is the Nash equilibrium. It's a set of actions based on a balanced strategy with characteristics that make it impossible for a single player to deviate from the set strategy and earn more rewards. In every case, the key point is how to find the best outcome among the strategies [25]. In the next part we introduced the way that the NSGA-II algorithm performs and finds Nash Equilibrium. The NSGA-II mathematical methods to acquire the NE which are based on non-cooperative matches include:

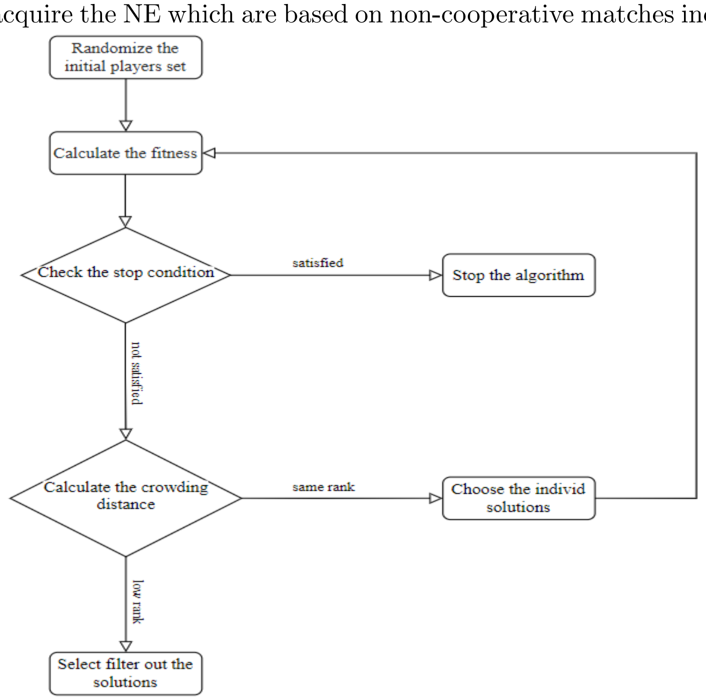

Figure 3. Procedure for obtaining the NE of a non-cooperative game in the NSGA-II algorithm

5.3 Formulas / parameters of NSGA-II algorithm

The initial code merely takes unrestricted problem into account, therefore it was improved to deal with limited difficulty. We cut down half of allocation points with possible solutions. To achieve those solutions, a number of actions are randomly taken to the first layouts. Table 8 below [26] contains some parameters for NSGA-II.

Parameter	Value	Note
Crossover probability	0.9	(M is the amount of aim variables)
Mutation probability	1/M	
Dispersal index for limited operator	20	
Dispersal index for unlimited operator	20	
Inhabitant amount	100 or 320	
Generation	500	

Table 8. Parameters for NSGA-II

5.4 Diagram and Pseudocode of NSGA-II algorithm

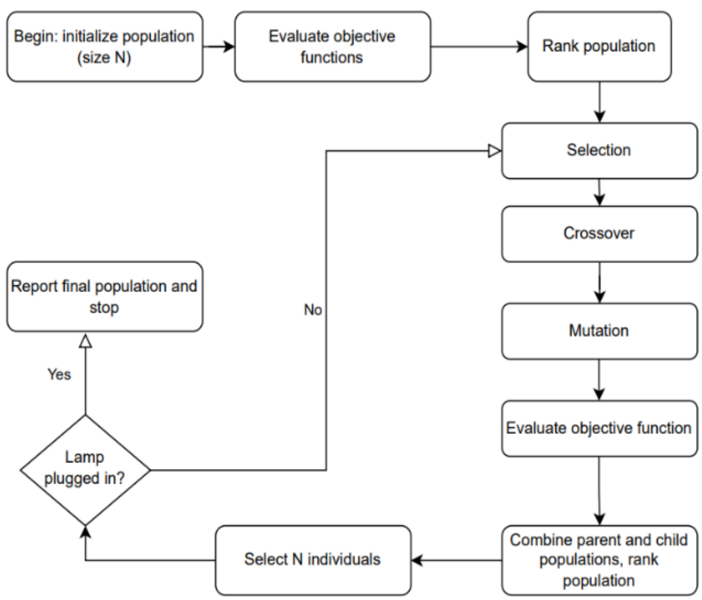

Figure 4. Pseudo code of NSGA-II in finding Nash equilibrium [27]

6 CONCLUSION

In this study, a game-theoretic strategy for resource distribution that is fair and efficient in the face of many emergencies is given. This method of solving this problem is an effective way to balance resources from relief areas to emergency locations. Our approach is to calculate the distances and materials of each region to find benefits and fairness for all emergency locations. The Nash equilibrium is proposed to be attained using an enhanced NSGA-II algorithm. The model and algorithm's usefulness and viability are demonstrated by the simulation results. The sea level rise causing many areas to be flooded is happening at a rapid rate. This study helps to strengthen methods for allocating resources for timely relief to damaged areas, thereby helping to reduce time and damage to the nation.

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